RACIONALIZACIJA

1. Racionaliziraj imenovalce.

a) $\frac{15}{\sqrt{5}}$

b) $\frac{13}{\sqrt{13}}$

c) $\frac{14}{\sqrt{6}}$

d) $\frac{\sqrt{35}}{\sqrt{5}}$

e) $\frac{1}{9\sqrt{2}}$

f) $\frac{\sqrt{6}}{2\sqrt{66}}$

g) $\frac{\sqrt{3}}{5\sqrt{24}}$

h) $\frac{45}{2\sqrt{15}}$

2. Racionaliziraj.

a) $\frac{2+\sqrt{3}}{\sqrt{3}}$

b) $\frac{2\sqrt{11}-44}{2\sqrt{11}}$

c) $\frac{4\sqrt{3}+3\sqrt{8}}{2\sqrt{42}}$

d) $\frac{5\sqrt{5}+7\sqrt{7}}{2\sqrt{35}}$

e) $\frac{2\sqrt{5}+20\sqrt{3}}{4\sqrt{15}}$

f) $\frac{7\sqrt{3}+3\sqrt{7}-\sqrt{21}}{\sqrt{21}}$

znamzasi

3. Izračunaj.

a) $\frac{59}{\sqrt{10}} - \frac{29}{\sqrt{10}}$

b) $\frac{7}{2\sqrt{3}} - \frac{1}{8\sqrt{3}}$

c) $\frac{\sqrt{6}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{6}}$

d) $\frac{\sqrt{3}}{\sqrt{5}} + \frac{3}{\sqrt{15}}$

4. Racionaliziraj.

a) $\frac{1}{\sqrt{3}+1}$

b) $\frac{\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

c) $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{15}}$

d) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

e) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

f) $\frac{3+4\sqrt{2}}{6-\sqrt{2}}$

g) $\frac{\sqrt{2}}{3-\sqrt{7}}$

5. Izračunaj.

a) $\frac{1}{13-2\sqrt{42}} - \frac{1}{13+2\sqrt{42}}$

b) $\frac{2\sqrt{3}}{\sqrt{5}+3} - \frac{\sqrt{5}}{2\sqrt{3}-4}$

c) $\frac{\sqrt{12-6\sqrt{3}}}{3-\sqrt{3}} - \frac{\sqrt{5}}{\sqrt{5}-2} + (\sqrt{5}+3)^2$

d) $(2+\sqrt{2})^3 - 3\sqrt{50} + \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}(5-\sqrt{24}) + \left(\sqrt{4+3\sqrt{2}} + \sqrt{3\sqrt{2}-4}\right)^2$

e) $(1+\sqrt{3})^3 - (1+\sqrt{3})^2 - \frac{2}{1+\sqrt{3}}$

znamzasi

6. Racionaliziraj.

a) $\frac{1}{\sqrt{a}-\sqrt{b}}$

b) $\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

c) $\frac{a+2\sqrt{ab}+4b}{\sqrt{a}+2\sqrt{b}}$

7. Racionaliziraj.

a) $\frac{46}{3-\sqrt{2+\sqrt{3}}}$

b) $\frac{\sqrt{6}-\sqrt{2}-\sqrt{3}}{2+\sqrt{2}-\sqrt{3}}$

8. Racionaliziraj.

a) $\frac{2}{\sqrt[4]{25}-\sqrt[4]{9}}$

b) $\frac{1}{\sqrt[6]{27}-\sqrt[6]{8}}$

c) $\frac{1}{\sqrt[3]{1-\sqrt{2}}}$

d) $\frac{1}{\sqrt[4]{2}-1}$

znamzasi

9. Racionaliziraj.

a) $\frac{\sqrt{a}-\sqrt{b}}{\sqrt[4]{a}-\sqrt[4]{b}}$

b) $\frac{a+b}{\sqrt[3]{a}+\sqrt[3]{b}}$

c) $\frac{\sqrt[3]{a}-\sqrt[3]{b}}{\sqrt[6]{a}-\sqrt[6]{b}}$

10. Dan je izraz $\frac{a+\sqrt{2}b}{a\sqrt{2}+b}$. Izračunaj njegovo vrednost za $a=(1-\sqrt{2})^3$ in $b=(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})$.

REŠITVE

1. a) $3\sqrt{5}$ b) $\sqrt{13}$ c) $\frac{7\sqrt{6}}{3}$ d) $\sqrt{7}$ e) $\frac{\sqrt{2}}{18}$

f) $\frac{\sqrt{11}}{22}$ g) $\frac{\sqrt{2}}{20}$ R) $\frac{3\sqrt{15}}{2}$

2. a) $\frac{3+2\sqrt{3}}{3}$ b) $1-2\sqrt{11}$ c) $\frac{\sqrt{21}+\sqrt{14}}{7}$ d) $\frac{25\sqrt{7}+49\sqrt{5}}{70}$

e) $\frac{6\sqrt{5}+\sqrt{3}}{6}$ f) $\sqrt{7}+\sqrt{3}-1$

3. a) $3\sqrt{10}$ b) $\frac{9\sqrt{3}}{8}$ c) $\frac{3\sqrt{2}}{2}$ d) $\frac{2\sqrt{15}}{5}$

4. a) $\frac{-1+\sqrt{3}}{2}$ b) $\frac{-1+\sqrt{3}}{2}$ c) $\frac{\sqrt{15}-5\sqrt{3}+3\sqrt{5}-3}{12}$ d) $5+2\sqrt{6}$

e) $-5-2\sqrt{6}$ f) $\frac{26+27\sqrt{2}}{34}$ g) $\frac{\sqrt{14}+3\sqrt{2}}{2}$

5. a) $4\sqrt{42}$ b) $\frac{3\sqrt{3}+2\sqrt{5}}{2}$ c) $10+4\sqrt{5}$ d) $19+7\sqrt{2}$ e) $7+3\sqrt{3}$

6. a) $\frac{\sqrt{a}+\sqrt{b}}{a-b}$ b) $a+\sqrt{a^2-1}$ c) $\frac{a\sqrt{a}-8b\sqrt{b}}{a-4b}$

7. a) $(7+\sqrt{3})(3+\sqrt{2+\sqrt{3}})$ b) $1-\sqrt{2}$

8. a) $\sqrt{5}+\sqrt{3}$ b) $\sqrt{3}+\sqrt{2}$ c) $-\sqrt[3]{1+\sqrt{2}}$ d) $1+\sqrt{2}+\sqrt[4]{2}+\sqrt[4]{8}$

9. a) $\sqrt[4]{a}+\sqrt[4]{b}$ b) $\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}$ c) $\sqrt[6]{a}+\sqrt[6]{b}$

10. $\frac{7+13\sqrt{2}}{17}$

REŠITVE PO KORAKIH

1. Racionaliziraj imenovalce.

a) $\frac{15}{\sqrt{5}} = \frac{15 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\overset{3}{\cancel{15}}\sqrt{5}}{\cancel{5}_{1}} = 3\sqrt{5}$

množiš s $\sqrt{5}$
obvezno SPODAJ IN ZGORAJ

Zgoraj in spodaj množiš z istim korenom.

$\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$ PRIMERI

$\sqrt{a} \cdot \sqrt{a} = a$
$\sqrt{3} \cdot \sqrt{3} = 3$
$\sqrt{2} \cdot \sqrt{2} = 2$

b) $\frac{13}{\sqrt{13}} = \frac{13 \cdot \sqrt{13}}{\sqrt{13} \cdot \sqrt{13}} = \frac{\overset{1}{\cancel{13}}\sqrt{13}}{\cancel{13}_{1}} = \sqrt{13}$

c) $\frac{14}{\sqrt{6}} = \frac{14 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{\overset{7}{\cancel{14}}\sqrt{6}}{\cancel{6}_{3}} = \frac{7\sqrt{6}}{3}$

delno korenиš

d) $\frac{\sqrt{35}}{\sqrt{5}} = \frac{\sqrt{35} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{35 \cdot 5}}{5} = \frac{\sqrt{7 \cdot 5 \cdot 5}}{5} = \frac{\sqrt{7 \cdot 25}}{5} = \frac{\overset{1}{\cancel{5}}\sqrt{7}}{\cancel{5}_{1}} = \sqrt{7}$

zgoraj in spodaj
množiš s $\sqrt{5}$ (s tistim korenom, ki je v imenovalcu)

znamzasi

e) $\frac{1}{9\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{9\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{9 \cdot 2} = \frac{\sqrt{2}}{18}$

množiš
s $\sqrt{2}$ (z 9 mi treba)

delno korenиš

f) $\frac{\sqrt{6}}{2\sqrt{66}} = \frac{\sqrt{6} \cdot \sqrt{66}}{2\sqrt{66} \cdot \sqrt{66}} = \frac{\sqrt{6 \cdot 66}}{2 \cdot 66} = \frac{\sqrt{6 \cdot 6 \cdot 11}}{2 \cdot 66} = \frac{\sqrt{36 \cdot 11}}{2 \cdot 66} =$

$= \frac{\overset{1}{\cancel{6}}\sqrt{11}}{2 \cdot \cancel{66}_{11}} = \frac{\sqrt{11}}{22}$

g) $\frac{\sqrt{3}}{5\sqrt{24}} = \frac{\sqrt{3}\cdot\sqrt{24}}{5\sqrt{24}\sqrt{24}} = \frac{\sqrt{3\cdot 24}}{5\cdot 24} = \frac{\sqrt{72}}{5\cdot 24} = \frac{\sqrt{36\cdot 2}}{5\cdot 24} = \frac{1\,6\sqrt{2}}{5\cdot 24_4} = \frac{\sqrt{2}}{20}$

delno korenis

h) $\frac{45}{2\sqrt{15}} = \frac{45\cdot\sqrt{15}}{2\sqrt{15}\cdot\sqrt{15}} = \frac{\overset{3}{45}\sqrt{15}}{2\cdot\cancel{15}_1} = \frac{3\sqrt{15}}{2}$

znamzasi

2. Racionaliziraj.

a) $\frac{2+\sqrt{3}}{\sqrt{3}} = \frac{(2+\sqrt{3})\cdot\sqrt{3}}{\sqrt{3}\sqrt{3}} =$ oklepaj je obvezen, ker množiš oba s $\sqrt{3}$

$= \frac{2\sqrt{3}+\sqrt{3}\cdot\sqrt{3}}{3} = \frac{2\sqrt{3}+3}{3}$

b) $\frac{2\sqrt{11}-44}{2\sqrt{11}} = \frac{(2\sqrt{11}-44)\sqrt{11}}{2\sqrt{11}\cdot\sqrt{11}} = \frac{2\sqrt{11}\cdot\sqrt{11}-44\sqrt{11}}{2\cdot 11} =$

$= \frac{2\cdot 11-44\sqrt{11}}{22} = \frac{22-44\sqrt{11}}{22} = \frac{\overset{1}{\cancel{22}}(1-2\sqrt{11})}{\cancel{22}_1} = 1-2\sqrt{11}$

izpostaviš 22, da lahko krajšaš

c) $\frac{4\sqrt{3}+3\sqrt{8}}{2\sqrt{42}} = \frac{(4\sqrt{3}+3\sqrt{8})\cdot\sqrt{42}}{2\sqrt{42}\cdot\sqrt{42}} = \frac{4\sqrt{3\cdot 42}+3\sqrt{8\cdot 42}}{2\cdot 42} =$ razstaviš, da lahko delno korenis

$3\cdot 42 = 3\cdot 2\cdot 21 = 3\cdot 2\cdot 3\cdot 7 = \underline{9}\cdot 14$
$8\cdot 42 = 2\cdot 4\cdot 2\cdot 21 = 2\cdot 4\cdot 2\cdot 3\cdot 7 = 4\cdot 4\cdot 3\cdot 7 = \underline{16}\cdot 21$

lahko korenis

$= \frac{4\sqrt{9\cdot 14}+3\sqrt{16\cdot 21}}{2\cdot 42} = \frac{4\cdot 3\sqrt{14}+3\cdot 4\sqrt{21}}{84} = \frac{12\sqrt{14}+12\sqrt{21}}{84} =$

$= \frac{\overset{1}{\cancel{12}}(\sqrt{14}+\sqrt{21})}{\cancel{84}_7} = \frac{\sqrt{14}+\sqrt{21}}{7}$

d) $\frac{5\sqrt{5}+7\sqrt{7}}{2\sqrt{35}} = \frac{(5\sqrt{5}+7\sqrt{7})\cdot\sqrt{35}}{2\sqrt{35}\cdot\sqrt{35}} = \frac{5\sqrt{5\cdot35}+7\sqrt{7\cdot35}}{2\cdot35} =$

$= \frac{5\sqrt{5\cdot5\cdot7}+7\sqrt{7\cdot7\cdot5}}{70} = \frac{5\cdot5\sqrt{7}+7\cdot7\sqrt{5}}{70} = \underline{\underline{\frac{25\sqrt{7}+49\sqrt{5}}{70}}}$

e) $\frac{2\sqrt{5}+20\sqrt{3}}{4\sqrt{15}} = \frac{(2\sqrt{5}+20\sqrt{3})\cdot\sqrt{15}}{4\sqrt{15}\cdot\sqrt{15}} = \frac{2\sqrt{5\cdot15}+20\sqrt{3\cdot15}}{4\cdot15} =$

$= \frac{2\sqrt{5\cdot3\cdot5}+20\sqrt{3\cdot3\cdot5}}{60} = \frac{2\cdot5\sqrt{3}+20\cdot3\sqrt{5}}{60} =$

$= \frac{10\sqrt{3}+60\sqrt{5}}{60} = \frac{\cancel{10}^{1}(\sqrt{3}+6\sqrt{5})}{\cancel{60}_{6}} = \underline{\underline{\frac{\sqrt{3}+6\sqrt{5}}{6}}}$

f) $\frac{7\sqrt{3}+3\sqrt{7}-\sqrt{21}}{\sqrt{21}} = \frac{(7\sqrt{3}+3\sqrt{7}-\sqrt{21})\sqrt{21}}{\sqrt{21}\cdot\sqrt{21}} =$

$= \frac{7\sqrt{3\cdot21}+3\sqrt{7\cdot21}-\sqrt{21\cdot21}}{21} = \frac{7\sqrt{3\cdot3\cdot7}+3\sqrt{7\cdot3\cdot7}-21}{21} =$

$= \frac{7\cdot3\sqrt{7}+3\cdot7\sqrt{3}-21}{21} = \frac{21\sqrt{7}+21\sqrt{3}-21}{21} = \frac{21(\sqrt{7}+\sqrt{3}-1)}{21} =$

$= \underline{\underline{\sqrt{7}+\sqrt{3}-1}}$

znamzasi

3. Izračunaj.

a) $\frac{59}{\sqrt{10}} - \frac{29}{\sqrt{10}} = \frac{59-29}{\sqrt{10}} =$

$= \frac{30\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = \frac{\cancel{30}^{3}\sqrt{10}}{\cancel{10}_{1}} = \underline{\underline{3\sqrt{10}}}$

Lahko najprej odsteješ in nato racionaliziraš ali obratno.

b) $\frac{7}{2\sqrt{3}} - \frac{1}{8\sqrt{3}} = \frac{7\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} - \frac{1\cdot\sqrt{3}}{8\sqrt{3}\cdot\sqrt{3}} =$

najprej racionaliziraš
(lahko daš tudi na skupni imenovalec in najprej odšteješ)

$= \frac{7\sqrt{3}}{2\cdot 3} - \frac{\sqrt{3}}{8\cdot 3} = \frac{7\sqrt{3}\cdot 4}{6\cdot 4} - \frac{\sqrt{3}}{24} = \frac{28\sqrt{3}}{24} - \frac{1\sqrt{3}}{24} =$

$= \frac{28\sqrt{3} - 1\sqrt{3}}{24} = \frac{\overset{9}{\cancel{27}}\sqrt{3}}{\cancel{24}_{8}} = \frac{9\sqrt{3}}{8}$

c) $\frac{\sqrt{6}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{6}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} + \frac{\sqrt{3}\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} = \frac{\sqrt{18}\cdot 2}{3\cdot 2} + \frac{\sqrt{18}}{6} = \frac{2\sqrt{18}}{6} + \frac{\sqrt{18}}{6} =$

$= \frac{\overset{1}{\cancel{3}}\sqrt{18}}{\cancel{6}\,2} = \frac{\sqrt{18}}{2} = \frac{\sqrt{9\cdot 2}}{2} = \frac{3\sqrt{2}}{2}$

d) $\frac{\sqrt{3}}{\sqrt{5}} + \frac{3}{\sqrt{15}} = \frac{\sqrt{3}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} + \frac{3\cdot\sqrt{15}}{\sqrt{15}\cdot\sqrt{15}} = \frac{\sqrt{15}}{5} + \frac{\overset{1}{\cancel{3}}\sqrt{15}}{\cancel{15}\,5} =$

$= \frac{\sqrt{15}}{5} + \frac{\sqrt{15}}{5} = \frac{2\sqrt{15}}{5}$

4. Racionaliziraj.

a) $\frac{1}{\sqrt{3}+1} = \frac{1\,(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} =$

$(a+b)(a-b) = a^2 - b^2$

nasprotni predznak
OKLEPAJ JE OBVEZEN

$= \frac{\sqrt{3}-1}{\sqrt{3}^2 - 1^2} = \frac{\sqrt{3}-1}{3-1} = \frac{\sqrt{3}-1}{2}$

b) $\frac{\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{\sqrt{2}\,(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \frac{\sqrt{2}\cdot\sqrt{6}-\sqrt{2}\cdot\sqrt{2}}{\sqrt{6}^2-\sqrt{2}^2} =$

$= \frac{\sqrt{12}-2}{6-2} = \frac{\sqrt{4\cdot 3}-2}{4} = \frac{2\sqrt{3}-2}{4} = \frac{{}^{1}\cancel{2}(\sqrt{3}-1)}{\cancel{4}\,2} = \underline{\underline{\frac{\sqrt{3}-1}{2}}}$

c) $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{15}} = \frac{(\sqrt{3}-\sqrt{5})(\sqrt{3}-\sqrt{15})}{(\sqrt{3}+\sqrt{15})(\sqrt{3}-\sqrt{15})} = \frac{3-\sqrt{45}-\sqrt{15}+\sqrt{75}}{\sqrt{3}^2-\sqrt{15}^2} =$

$= \frac{3-\sqrt{9\cdot 5}-\sqrt{15}+\sqrt{25\cdot 3}}{3-15} = \frac{3-3\sqrt{5}-\sqrt{15}+5\sqrt{3}}{-12} =$

minus dam gor (vsem spremenim predznake)

$= \underline{\underline{\frac{-3+3\sqrt{5}+\sqrt{15}-5\sqrt{3}}{12}}}$

d) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} =$

$= \frac{9\cdot 2+6\sqrt{6}+6\sqrt{6}+4\cdot 3}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \frac{18+12\sqrt{6}+12}{9\cdot 2-4\cdot 3} = \frac{30+12\sqrt{6}}{18-12} =$

$= \frac{{}^{1}\cancel{6}(5+2\sqrt{6})}{\cancel{6}\,1} = \underline{\underline{5+2\sqrt{6}}}$

e) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{2+\sqrt{6}+\sqrt{6}+3}{\sqrt{2}^2-\sqrt{3}^2} =$

$= \frac{5+2\sqrt{6}}{2-3} = \frac{5+2\sqrt{6}}{-1} = \underline{\underline{-5-2\sqrt{6}}}$

f) $\frac{3+4\sqrt{2}}{6-\sqrt{2}} = \frac{(3+4\sqrt{2})(6+\sqrt{2})}{(6-\sqrt{2})(6+\sqrt{2})} = \frac{18+3\sqrt{2}+24\sqrt{2}+4\cdot 2}{6^2-\sqrt{2}^2} =$

$= \frac{18+27\sqrt{2}+8}{36-2} = \underline{\underline{\frac{26+27\sqrt{2}}{34}}}$

g) $\frac{\sqrt{2}}{3-\sqrt{7}} = \frac{\sqrt{2}\,(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{3\sqrt{2}+\sqrt{14}}{3^2-\sqrt{7}^2} = \frac{3\sqrt{2}+\sqrt{14}}{9-7} =$

$= \underline{\underline{\frac{3\sqrt{2}+\sqrt{14}}{2}}}$

5. Izračunaj.

a) $\frac{1}{13-2\sqrt{42}} - \frac{1}{13+2\sqrt{42}} = \frac{1\,(13+2\sqrt{42})}{(13-2\sqrt{42})(13+2\sqrt{42})} - \frac{1\,(13-2\sqrt{42})}{(13+2\sqrt{42})(13-2\sqrt{42})} =$

najprej racionaliziraš imenovalce

$= \frac{13+2\sqrt{42}}{13^2-(2\sqrt{42})^2} - \frac{13-2\sqrt{42}}{13^2-(2\sqrt{42})^2} = \frac{13+2\sqrt{42}}{169-4\cdot 42} - \frac{13-2\sqrt{42}}{169-4\cdot 42} =$

$= \frac{13+2\sqrt{42}}{1} - \frac{13-2\sqrt{42}}{1} = 13+2\sqrt{42}-13+2\sqrt{42} = \underline{\underline{4\sqrt{42}}}$

minus pred ulomkom spremeni predznak VSEM v števcu

b) $\frac{2\sqrt{3}}{\sqrt{5}+3} - \frac{\sqrt{5}}{2\sqrt{3}-4} = \frac{2\sqrt{3}\,(\sqrt{5}-3)}{(\sqrt{5}+3)(\sqrt{5}-3)} - \frac{\sqrt{5}\,(2\sqrt{3}+4)}{(2\sqrt{3}-4)(2\sqrt{3}+4)} =$

$= \frac{2\sqrt{15}-6\sqrt{3}}{\sqrt{5}^2-3^2} - \frac{2\sqrt{15}+4\sqrt{5}}{4\cdot 3-16} = \frac{2\sqrt{15}-6\sqrt{3}}{5-9} - \frac{2\sqrt{15}+4\sqrt{5}}{-4} =$

$= \frac{2\sqrt{15}-6\sqrt{3}}{-4} - \frac{2\sqrt{15}+4\sqrt{5}}{-4} = \frac{2\sqrt{15}-6\sqrt{3}-2\sqrt{15}-4\sqrt{5}}{-4} =$

$= \frac{-6\sqrt{3}-4\sqrt{5}}{-4} = \frac{-2(3\sqrt{3}+2\sqrt{5})}{-4_2} = \underline{\underline{\frac{3\sqrt{3}+2\sqrt{5}}{2}}}$

če izpostaviš minus, se vsem v oklepaju spremenijo predznaki

c) $\frac{\sqrt{12-6\sqrt{3}}}{3-\sqrt{3}} - \frac{\sqrt{5}}{\sqrt{5}-2} + (\sqrt{5}+3)^2 =$

Korenis

$\sqrt{12-6\sqrt{3}} = a-b$

$12-6\sqrt{3} = (a-b)^2$

$12-6\sqrt{3} = a^2-2ab+b^2$

a^2+b^2 $\quad -2ab$

$-2ab = -6\sqrt{3} \;/:(-2)$

$ab = 3\sqrt{3}$

$a=1 \quad b=3\sqrt{3}$

$a=3 \quad b=\sqrt{3}$

$a^2+b^2 = 12$

$1+9\cdot 3 = 28$

$3^2+\sqrt{3}^2 = 12$ ✓

znamzasi

$= \frac{3-\sqrt{3}}{3-\sqrt{3}} - \frac{\sqrt{5}(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} + \sqrt{5}^2 + 2\sqrt{5}\cdot 3 + 3^2 =$

$= 1 - \frac{5+2\sqrt{5}}{\sqrt{5}^2-2^2} + 5 + 6\sqrt{5} + 9 = 1 - \frac{5+2\sqrt{5}}{5-4} + 14 + 6\sqrt{5} =$ $\quad \frac{5+2\sqrt{5}}{5-4} = 1$

$= 1-5-2\sqrt{5}+14+6\sqrt{5} = \underline{\underline{10+4\sqrt{5}}}$

d) $(2+\sqrt{2})^3 - 3\sqrt{50} + \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}(5-\sqrt{24}) + \left(\sqrt{4+3\sqrt{2}} + \sqrt{3\sqrt{2}-4}\right)^2 =$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(2+\sqrt{2})^3 = 2^3+3\cdot 2^2\sqrt{2}+3\cdot 2\sqrt{2}^2+\sqrt{2}^3 = 8+12\sqrt{2}+6\cdot 2+2\sqrt{2} =$

$= 8+12\sqrt{2}+12+2\sqrt{2} = 20+14\sqrt{2}$

$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{2+\sqrt{6}+\sqrt{6}+3}{2-3} = \frac{5+2\sqrt{6}}{-1} =$

$= -5-2\sqrt{6}$

$(\sqrt{4+3\sqrt{2}}+\sqrt{3\sqrt{2}-4})^2 = \sqrt{4+3\sqrt{2}}^2 + 2\sqrt{(4+3\sqrt{2})(3\sqrt{2}-4)} + \sqrt{3\sqrt{2}-4}^2 =$

$= 4+3\sqrt{2} + 2\sqrt{12\sqrt{2}-16+9\cdot 2-12\sqrt{2}} + 3\sqrt{2} - 4 =$

$= 4+3\sqrt{2}+2\sqrt{-16+18}+3\sqrt{2}-4 = 3\sqrt{2}+2\sqrt{2}+3\sqrt{2} = 8\sqrt{2}$

$= 20+14\sqrt{2} - 3\sqrt{25\cdot 2} + (-5-2\sqrt{6})(5-\sqrt{24}) + 8\sqrt{2} =$

$= 20+22\sqrt{2} - 3\cdot 5\sqrt{2} - 25 + 5\sqrt{24} - 10\sqrt{6} + 2\sqrt{144} =$

$= 20+22\sqrt{2}-15\sqrt{2}-25+5\sqrt{4\cdot 6}-10\sqrt{6}+2\cdot 12 =$

$= -5+7\sqrt{2}+5\cdot 2\sqrt{6}-10\sqrt{6}+24 = \underline{\underline{19+7\sqrt{2}}}$

znamzasi

e) $(1+\sqrt{3})^3 - (1+\sqrt{3})^2 - \frac{2}{1+\sqrt{3}} =$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)^2 = a^2+2ab+b^2$

$(1+\sqrt{3})^3 = 1^3+3\cdot 1^2\sqrt{3}+3\cdot 1\sqrt{3}^2+\sqrt{3}^3 =$

$= 1+3\sqrt{3}+3\cdot 3+3\sqrt{3} = 10+6\sqrt{3}$

$(1+\sqrt{3})^2 = 1^2+2\sqrt{3}+\sqrt{3}^2 = 1+2\sqrt{3}+3 = 4+2\sqrt{3}$

$\frac{2}{1+\sqrt{3}} = \frac{2(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{2(1-\sqrt{3})}{1^2-\sqrt{3}^2} = \frac{2(1-\sqrt{3})}{1-3} = \frac{\overset{1}{\cancel{2}}(1-\sqrt{3})}{\cancel{-2}_{-1}} =$

$= -1+\sqrt{3}$

$= 10+6\sqrt{3}-(4+2\sqrt{3})-(-1+\sqrt{3}) = 10+6\sqrt{3}-4-2\sqrt{3}+1-\sqrt{3} =$

$= \underline{\underline{7+3\sqrt{3}}}$

6. Racionaliziraj.

a) $\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{1(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}^2-\sqrt{b}^2} = \underline{\underline{\frac{\sqrt{a}+\sqrt{b}}{a-b}}}$

b) $\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}} = \frac{(\sqrt{a+1}+\sqrt{a-1})(\sqrt{a+1}+\sqrt{a-1})}{(\sqrt{a+1}-\sqrt{a-1})(\sqrt{a+1}+\sqrt{a-1})} =$

$= \frac{a+1+\sqrt{(a+1)(a-1)}+\sqrt{(a-1)(a+1)}+a-1}{\sqrt{a+1}^2-\sqrt{a-1}^2} =$

$= \frac{a+1+2\sqrt{a^2-1}+a-1}{a+1-(a-1)} = \frac{2a+2\sqrt{a^2-1}}{a+1-a+1} = \frac{\overset{1}{\cancel{2}}(a+\sqrt{a^2-1})}{\underset{1}{\cancel{2}}} =$

$= \underline{\underline{a+\sqrt{a^2-1}}}$

znamzasi

c) $\frac{a+2\sqrt{ab}+4b}{\sqrt{a}+2\sqrt{b}} = \frac{(a+2\sqrt{ab}+4b)(\sqrt{a}-2\sqrt{b})}{(\sqrt{a}+2\sqrt{b})(\sqrt{a}-2\sqrt{b})} =$

$= \frac{a\sqrt{a}-2a\sqrt{b}+2\sqrt{ab}\sqrt{a}-4\sqrt{ab}\sqrt{b}+4b\sqrt{a}-8b\sqrt{b}}{\sqrt{a}^2-(2\sqrt{b})^2} =$

$= \frac{a\sqrt{a}-2a\sqrt{b}+2\sqrt{a^2b}-4\sqrt{ab^2}+4b\sqrt{a}-8b\sqrt{b}}{a-4b} =$

$= \frac{a\sqrt{a}-\cancel{2a\sqrt{b}}+\cancel{2a\sqrt{b}}-\cancel{4b\sqrt{a}}+\cancel{4b\sqrt{a}}-8b\sqrt{b}}{a-4b} = \underline{\underline{\frac{a\sqrt{a}-8b\sqrt{b}}{a-4b}}}$

7. Racionaliziraj.

a) $\frac{46}{3-\sqrt{2+\sqrt{3}}} = \frac{46(3+\sqrt{2+\sqrt{3}})}{(3-\sqrt{2+\sqrt{3}})(3+\sqrt{2+\sqrt{3}})} = \frac{46(3+\sqrt{2+\sqrt{3}})}{9-\sqrt{2+\sqrt{3}}^2} =$

$= \frac{46(3+\sqrt{2+\sqrt{3}})}{9-(2+\sqrt{3})} = \frac{46(3+\sqrt{2+\sqrt{3}})}{9-2-\sqrt{3}} = \frac{46(3+\sqrt{2+\sqrt{3}})}{7-\sqrt{3}} =$

$$= \frac{46\,(3+\sqrt{2+\sqrt{3}})(7+\sqrt{3})}{(7-\sqrt{3})(7+\sqrt{3})} = \frac{46(3+\sqrt{2+\sqrt{3}})(7+\sqrt{3})}{7^2-\sqrt{3}^2} =$$

$$= \frac{\overset{1}{\cancel{46}}(3+\sqrt{2+\sqrt{3}})(7+\sqrt{3})}{\cancel{46}\,1} = \underline{\underline{(3+\sqrt{2+\sqrt{3}})(7+\sqrt{3})}}$$

znamzasi

b) $\dfrac{\sqrt{6}-\sqrt{2}-\sqrt{3}}{2+\sqrt{2}-\sqrt{3}} = \dfrac{(\sqrt{6}-\sqrt{2}-\sqrt{3})(2+\sqrt{2}+\sqrt{3})}{(2+\sqrt{2}-\sqrt{3})(2+\sqrt{2}+\sqrt{3})} =$

$$= \frac{\cancel{2\sqrt{6}}+\sqrt{12}+\sqrt{18}-2\sqrt{2}-2-\cancel{\sqrt{6}}-2\sqrt{3}-\cancel{\sqrt{6}}-3}{4+2\sqrt{2}+\cancel{2\sqrt{3}}+2\sqrt{2}+2+\cancel{\sqrt{6}}-\cancel{2\sqrt{3}}-\cancel{\sqrt{6}}-3} =$$

$$= \frac{\sqrt{4\cdot 3}+\sqrt{9\cdot 2}-2\sqrt{2}-2-2\sqrt{3}-3}{3+4\sqrt{2}} = \frac{\cancel{2\sqrt{3}}+3\sqrt{2}-2\sqrt{2}-2-\cancel{2\sqrt{3}}-3}{3+4\sqrt{2}} =$$

$$= \frac{\sqrt{2}-5}{3+4\sqrt{2}} = \frac{(\sqrt{2}-5)(3-4\sqrt{2})}{(3+4\sqrt{2})(3-4\sqrt{2})} = \frac{3\sqrt{2}-4\cdot 2-15+20\sqrt{2}}{3^2-(4\sqrt{2})^2} =$$

$$= \frac{23\sqrt{2}-23}{9-16\cdot 2} = \frac{23\sqrt{2}-23}{-23} = \frac{\cancel{23}(\sqrt{2}-1)}{\cancel{-23}\,-1} = \frac{\sqrt{2}-1}{-1} = \underline{\underline{1-\sqrt{2}}}$$

8. Racionaliziraj.

a) $\dfrac{2}{\sqrt[4]{25}-\sqrt[4]{9}} = \dfrac{2}{\sqrt[4]{5^2}-\sqrt[4]{3^2}} = \dfrac{2}{\sqrt{5}-\sqrt{3}} =$

$\sqrt[\cancel{4}^{2}]{5^{\cancel{2}^{1}}} = \sqrt{5}$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} =$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3} = \frac{\cancel{2}(\sqrt{5}+\sqrt{3})}{\cancel{2}} = \underline{\underline{\sqrt{5}+\sqrt{3}}}$$

b) $\dfrac{1}{\sqrt[6]{27}-\sqrt[6]{8}} = \dfrac{1}{\sqrt[6]{3^3}-\sqrt[6]{2^3}} = \dfrac{1}{\sqrt{3}-\sqrt{2}} = \dfrac{1\,(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} =$

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2} = \frac{\sqrt{3}+\sqrt{2}}{1} = \underline{\underline{\sqrt{3}+\sqrt{2}}}$$

c) $\frac{1}{\sqrt[3]{1-\sqrt{2}}} = \frac{1\cdot\sqrt[3]{(1-\sqrt{2})^2}}{\sqrt[3]{1-\sqrt{2}}\,\sqrt[3]{(1-\sqrt{2})^2}} = \frac{\sqrt[3]{(1-\sqrt{2})^2}}{\sqrt[3]{(1-\sqrt{2})^3}} = \frac{\sqrt[3]{(1-\sqrt{2})^2}}{1-\sqrt{2}} =$

$= \frac{\sqrt[3]{(1-\sqrt{2})^2}\,(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{\sqrt[3]{(1-\sqrt{2})^2}\,(1+\sqrt{2})}{1-2} = \frac{\sqrt[3]{(1-\sqrt{2})^2}\,(1+\sqrt{2})}{-1} =$

$= -\sqrt[3]{(1-\sqrt{2})^2}\,(1+\sqrt{2}) = -\sqrt[3]{(1-\sqrt{2})^2}\cdot\sqrt[3]{(1+\sqrt{2})^3} =$

lahko tako pustiš in podčrtaš

$= -\sqrt[3]{(1-2\sqrt{2}+2)(1+3\sqrt{2}+3\cdot 2+2\sqrt{2})} = -\sqrt[3]{(3-2\sqrt{2})(7+5\sqrt{2})} =$

$= -\sqrt[3]{21+15\sqrt{2}-14\sqrt{2}-10\cdot 2} = \underline{\underline{-\sqrt[3]{1+\sqrt{2}}}}$

d) $\frac{1}{\sqrt[4]{2}-1} = \frac{1\,(\sqrt[4]{2}+1)}{(\sqrt[4]{2}-1)(\sqrt[4]{2}+1)} = \frac{\sqrt[4]{2}+1}{\sqrt{4}-1} = \frac{\sqrt[4]{2}+1}{\sqrt{2}-1} =$

$\sqrt[4]{4} = \sqrt[4]{2^2} = \sqrt{2}$

$= \frac{(\sqrt[4]{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt[4]{2}\cdot\sqrt[4]{2^{\cdot 2}}+\sqrt[4]{2}+\sqrt{2}+1}{\sqrt{2}^2-1} = \frac{\sqrt[4]{2}\cdot\sqrt[4]{2^2}+\sqrt[4]{2}+\sqrt{2}+1}{1} =$

$= \underline{\underline{\sqrt[4]{8}+\sqrt[4]{2}+\sqrt{2}+1}}$

znamzas5i

9. Racionaliziraj.

a) $\frac{\sqrt{a}-\sqrt{b}}{\sqrt[4]{a}-\sqrt[4]{b}} = \frac{(\sqrt{a}-\sqrt{b})(\sqrt[4]{a}+\sqrt[4]{b})}{(\sqrt[4]{a}-\sqrt[4]{b})(\sqrt[4]{a}+\sqrt[4]{b})} = \frac{(\sqrt{a}-\sqrt{b})(\sqrt[4]{a}+\sqrt[4]{b})}{(\sqrt[4]{a})^2-(\sqrt[4]{b})^2} =$

$= \frac{(\sqrt{a}-\sqrt{b})(\sqrt[4]{a}+\sqrt[4]{b})}{\sqrt{a}-\sqrt{b}} = \underline{\underline{\sqrt[4]{a}+\sqrt[4]{b}}}$

b) $\frac{a+b}{\sqrt[3]{a}+\sqrt[3]{b}} = \frac{(a+b)(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})}{(\sqrt[3]{a}+\sqrt[3]{b})(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})} =$

$a+b \qquad a^2-ab+b^2$

$= \frac{(a+b)(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})}{(\sqrt[3]{a})^3+(\sqrt[3]{b})^3} =$

$a^3+b^3=(a+b)(a^2-ab+b^2)$
$a^3-b^3=(a-b)(a^2+ab+b^2)$

$= \frac{\cancel{(a+b)}(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2})}{\cancel{a+b}} =$

$= \underline{\underline{\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}}}$

c) $\frac{\sqrt[3]{a}-\sqrt[3]{b}}{\sqrt[6]{a}-\sqrt[6]{b}} = \frac{(\sqrt[3]{a}-\sqrt[3]{b})(\sqrt[6]{a}+\sqrt[6]{b})}{(\sqrt[6]{a}-\sqrt[6]{b})(\sqrt[6]{a}+\sqrt[6]{b})} = \frac{(\sqrt[3]{a}-\sqrt[3]{b})(\sqrt[6]{a}+\sqrt[6]{b})}{(\sqrt[6]{a})^2-(\sqrt[6]{b})^2} =$

$= \frac{\cancel{(\sqrt[3]{a}-\sqrt[3]{b})}(\sqrt[6]{a}+\sqrt[6]{b})}{\cancel{\sqrt[3]{a}-\sqrt[3]{b}}} = \underline{\underline{\sqrt[6]{a}+\sqrt[6]{b}}}$

10. Dan je izraz $\frac{a+\sqrt{2}b}{a\sqrt{2}+b}$. Izračunaj njegovo vrednost za $a=(1-\sqrt{2})^3$ in $b=(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})$

$a=(1-\sqrt{2})^3 = 1-3\cdot 1^2\sqrt{2}+3\cdot 1\sqrt{2}^2-\sqrt{2}^3 = 1-3\sqrt{2}+6-2\sqrt{2} = 7-5\sqrt{2}$

$b=(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6}) = \sqrt{7}^2-\sqrt{6}^2 = 7-6 = 1$

$\frac{a+\sqrt{2}b}{a\sqrt{2}+b} = \frac{7-5\sqrt{2}+\sqrt{2}\cdot 1}{(7-5\sqrt{2})\sqrt{2}+1} = \frac{7-4\sqrt{2}}{7\sqrt{2}-5\cdot 2+1} = \frac{7-4\sqrt{2}}{-9+7\sqrt{2}} =$

$= \frac{(7-4\sqrt{2})(-9-7\sqrt{2})}{(-9+7\sqrt{2})(-9-7\sqrt{2})} = \frac{-63-49\sqrt{2}+36\sqrt{2}+28\cdot 2}{(-9)^2-(7\sqrt{2})^2} =$

$= \frac{-7-13\sqrt{2}}{81-49\cdot 2} = \frac{-7-13\sqrt{2}}{-17} \begin{matrix}\cdot(-1)\\ \cdot(-1)\end{matrix} = \underline{\underline{\frac{7+13\sqrt{2}}{17}}}$